SUPERSONIC AXISYMMETRIC CONICAL FLOWS WITH CONICAL SHOCKS ADJACENT TO UNIFORM PARALLEL FLOWS

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Busemann [1,2] has given the general theory of axisymmetric supersonic conical flows and examined in detail two classes of such flows: flow around a circular cone and flow in a compression diffusor ending in a conical shock. Nikolskii [3] has examined continuously expanding conical flows corresponding to flows around boat-tails of given shape. The present paper considers all the possible classes of supersonic axisymmetric conical flows with conical shock waves adjacent to uniform parallel flows. Two new types of conical flow are obtained: converging flows behind conical shocks and diverging flows in front of conical shocks. These correspond to flows in specially shaped channels (Figs. 2,4).

It is shown that at supersonic speeds there exist four classes of axisymmetric conical flows with conical shock waves adjacent to parallel uniform flows: diverging and converging flows upstream and downstream of conical shocks.

Four possible combinations of conical shocks and regions of uniform parallel velocity u_0 are represented in Fig. 1: diverging or converging conical shock with a free-stream velocity upstream or downstream of the shock.

We shall prove that to each such combination corresponds a special class of (continuous) conical flows, matching the shock wave. We can determine the boundary values of these conical flows, u_1, v_1 in the hodograph plane with the help of shock polars [4] (Fig. 1).

$$\left(\frac{dv}{du}\right)_1 = \frac{v_1}{u_1 - u_0} \tag{1}$$

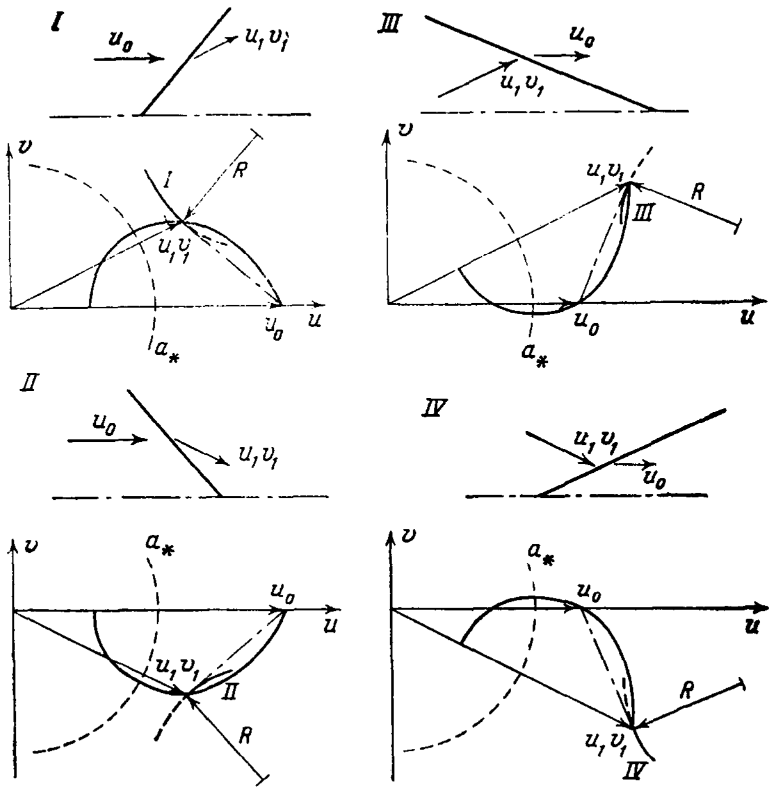

Fig. 1.

Axisymmetric conical flows are governed by the usual differential equation of second order [2]

$$v\frac{d^2v}{du^2} = 1 + \left(\frac{dv}{du}\right)^2 - \frac{2/(\varkappa+1)(u+v\,dv/du)^2}{a_*^2 - (u^2+v^2)(\varkappa-1)/(\varkappa+1)} \tag{2}$$

and by the following relation between the physical r, x plane and the hodograph u, v plane:

$$\frac{x}{r} = -\frac{dv}{du} \tag{3}$$

Equation (2) has singularities only on the axis and on the circle of the maximum speed W_{max} (corresponding to expansion to vacuum). Therefore, through every initial point (u_1, v_1) with a given inclination (1) there passes just one integral curve of Equation (2). The radius of curvature of these curves in the vicinity of the point (u_1, v_1) are shown in Fig. 1.

Only one side of the integral curve (shown unbroken in Fig. 1) corresponds to the physically sensible case where the conical flow does not penetrate into the uniform flow u_0. The four types of integral curves *I, II, III,* and *IV* (Fig. 1) determine the four possible classes of conical flows with conical shocks, adjacent to the parallel flow u_0.

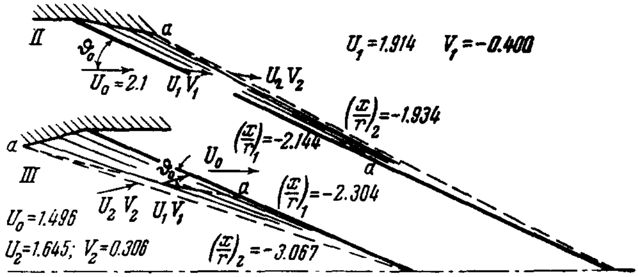

Fig. 2.

Cases *I* and *IV* were examined by Busemann [1, 2]. Flows of class *II* - converging flows behind conical shocks and class *III* - diverging conical flows upstream of conical shocks appear to be new. The integral curves of flows *II* and *III* proceed from the initial point (u_1, v_1) to the endpoint (u_2, v_2), where $d^2v/du^2 = 0$. In the physical plane, the continuous conical flows are bounded by the conical shock, the limiting characteristic (corresponding to u_2, v_2) and the flow boundaries. Examples of flows *II* and *III* are displayed in Figs. 2 and 3 and in Tables 1-5. In the figures

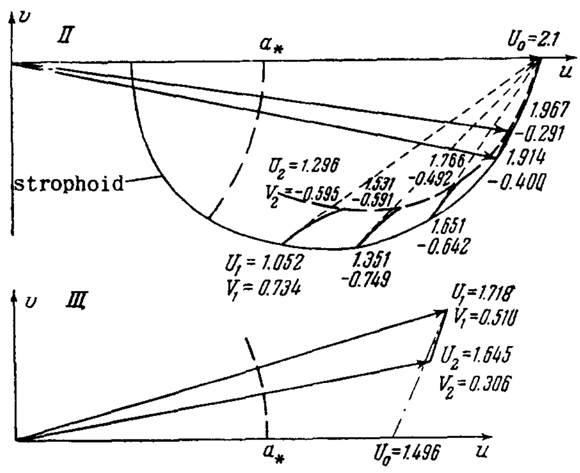

Fig. 3.

TABLE 1.

Flow *II*, $\lambda_0 = 2.1$, $\vartheta_0 = 25°$

x/r	u/a_*	v/a_*	λ	M	α	ϑ	$\overline{r_s}$
—2.144	1.914	—0.400	1.955	2.960	19°46	—11°48	1.000
—2.129	1.916	—0.395	1.956	2.970	19°42	—11°40	1.030
—2.114	1.918	—0.391	1.957	2.971	19°42	—11°32	1.061
—2.100	1.920	—0.387	1.958	2.975	19°38	—11°24	1.094
—2.087	1.922	—0.383	1.959	2.979	19°38	—11°16	1.128
—2.074	1.924	—0.379	1.960	2.980	19°38	—11°08	1.164
—2.062	1.926	—0.374	1.962	2.992	19°33	—11°00	1.200
—2.051	1.928	—0.370	1.963	2.996	19°30	—10°52	1.240
—2.040	1.930	—0.366	1.964	3.000	19°30	—10°45	1.278
—2.030	1.932	—0.362	1.965	3.004	19°29	—10°37	1.321
—2.020	1.934	—0.358	1.966	3.009	19°28	—10°30	1.364
—2.010	1.936	—0.354	1.968	3.010	19°28	—10°22	1.409
—2.002	1.938	—0.350	1.969	3.022	19°20	—10°14	1.457
—1.994	1.940	—0.346	1.970	3.026	19°17	—10°07	1.506
—1.986	1.942	—0.342	1.971	3.035	19°15	— 9°59	1.559
—1.980	1.944	—0.338	1.973	3.039	19°13	— 9°57	1.613
—1.973	1.946	—0.334	1.974	3.044	19°12	— 9°45	1.671
—1.967	1.948	—0.330	1.975	3.048	19°09	— 9°37	1.731
—1.962	1.950	—0.326	1.977	3.050	19°09	— 9°30	1.792
—1.958	1.952	—0.322	1.978	3.061	19°06	— 9°23	1.856
—1.954	1.954	—0.318	1.979	3.066	19°03	— 9°15	1.928
—1.950	1.956	—0.314	1.981	3.075	18°58	— 9°08	1.999
—1.947	1.958	—0.310	1.982	3.079	18°57	— 9°01	2.075
—1.945	1.960	—0.307	1.983	3.081	18°56	— 8°54	2.153
—1.943	1.962	—0.303	1.985	3.093	18°54	— 8°47	2.241
—1.941	1.964	—0.299	1.986	3.097	18°51	— 8°39	2.329
—1.938	1.966	—0.295	1.988	3.100	18°50	— 8°32	2.416

TABLE 2.

Flow *II*, $\lambda_0 = 2.1$, $\vartheta_0 = 35°$

x/r	u/a_*	v/a_*	λ	M	α	ϑ	$\overline{r_s}$
—1.428	1.651	—0.642	1.772	2.340	25°18	—21°15	1.0
—1.415	1.655	—0.636	1.773	2.346	25°13	—21°02	1.010
—1.402	1.659	—0.630	1.774	2.349	25°11	—20°49	1.039
—1.390	1.663	—0.625	1.776	2.354	25°09	—20°36	1.067
—1.377	1.667	—0.619	1.778	2.360	25°06	—20°23	1.097
—1.366	1.671	—0.614	1.779	2.365	25°03	—20°11	1.130
—1.355	1.675	—0.608	1.782	2.371	24°58	—19°58	1.164
—1.345	1.679	—0.603	1.784	2.377	24°54	—19°46	1.197
—1.335	1.683	—0.598	1.786	2.379	24°53	—19°33	1.233
—1.326	1.687	—0.592	1.787	2.385	24°47	—19°21	1.269
—1.317	1.691	—0.587	1.789	2.391	24°43	—19°09	1.310
—1.309	1.695	—0.582	1.791	2.399	24°39	—18°57	1.349
—1.301	1.699	—0.576	1.794	2.405	24°32	—18°46	1.396
—1.289	1.706	—0.567	1.798	2.414	24°28	—18°24	1.442
—1.273	1.716	—0.555	1.803	2.432	24°20	—17°55	1.570
—1.260	1.726	—0.542	1.809	2.449	24°04	—17°26	1.706
—1.249	1.736	—0.529	1.814	2.464	24°0	—16°58	1.853
—1.242	1.746	—0.517	1.820	2.482	23°47	—16°30	2.0223
—1.237	1.756	—0.504	1.826	2.501	23°35	—16°02	2.211
—1.235	1.764	—0.494	1.831	2.518	23°27	—15°38	2.420

TABLE 3.

Flow *II*, $\lambda_0 = 2.1$, $\vartheta_0 = 45°$

x/r	u/a_*	v/a_*	λ	M	α	ϑ	$\overline{r_2}$
—1.000	1.351	—0.749	1.545	1.818	33°23	—29°	1.0
—0.992	1.355	—0.745	1.546	1.819	33°23	—28°48	1.016
—0.984	1.359	—0.741	1.548	1.823	33°16	—28°36	1.034
—0.973	1.365	—0.735	1.551	1.827	33°10	—28°18	1.0611
—0.962	1.371	—0.729	1.553	1.833	33°04	—28°01	1.086
—0.951	1.377	—0.724	1.556	1.839	32°58	—27°43	1.118
—0.941	1.383	—0.718	1.558	1.843	32°51	—27°26	1.148
—0.930	1.390	—0.711	1.562	1.849	32°45	—27°06	1.184
—0.917	1.398	—0.704	1.565	1.857	32°36	—26°44	1.228
—0.906	1.406	—0.697	1.569	1.865	32°25	—26°21	1.275
—0.895	1.414	—0.689	1.573	1.873	32°17	—25°59	1.322
—0.882	1.424	—0.680	1.578	1.883	32°04	—25°33	1.3856
—0.870	1.434	—0.672	1.584	1.896	31°50	—25°05	1.4553
—0.860	1.444	—0.663	1.589	1.906	31°36	—24°40	1.530
—0,850	1.454	—0.654	1.595	1.919	31°28	—24°14	1.609
—0.842	1.464	—0.646	1.600	1.929	31°14	—23°48	1.690
—0.835	1.474	—0.637	1.606	1.942	31°0	—23°23	1.781
—0.829	1.484	—0.629	1.612	1.954	30°48	—22°59	1.849
—0.824	1.494	—0.621	1.618	1.967	30°32	—22°34	1.984
—0.820	1.504	—0.613	1.624	1.980	30°20	—22°10	2.097
—0.818	1.514	—0.604	1.630	1.993	30°10	—21°47	2.215
—0.816	1.524	—0.596	1.637	2.009	29°50	—21°22	2.355
—0.816	1.531	—0.591	1.641	2.018	29°44	—21°06	2.451

TABLE 4.

Flow *II*, $\lambda_0 = 2.1$, $\vartheta_0 = 55°$

x/r	u/a_*	v/a_*	λ	M	α	ϑ	$\overline{r_2}$
—0.700	1.052	—0.734	1.283	1.375	46°39	—34°54	1.000
—0.684	1.062	—0.727	1.288	1.381	46°23	—34°23	1.0295
—0.670	1.072	—0.720	1.291	1.387	46°13	—33°53	1.060
—0.655	1.082	—0.713	1.297	1.394	45°51	—33°23	1.092
—0.641	1.092	—0.707	1.301	1.402	45°34	—32°55	1.125
—0.628	1.102	—0.700	1.306	1.409	45°15	—32°26	1.160
—0.616	1.112	—0.694	1.311	1.417	44°55	—31°58	1.196
—0.604	1.122	—0.688	1.316	1.424	44°38	—31°31	1.234
—0.593	1 132	—0.682	1.322	1.432	44°12	—31°04	1.274
—0.582	1.142	—0.676	1.327	1.441	43°57	—30°38	1.316
—0.572	1.152	—0.670	1.333	1.450	43°38	—30°13	1.359
—0.563	1.162	—0.665	1.339	1.459	43°17	—29°46	1.406
—0.555	1.172	—0.659	1.345	1.469	42°55	—29°21	1.455
—0.546	1.183	—0.653	1.351	1.479	42°32	—28°54	1.511
—0.537	1.195	—0.646	1.359	1.491	42°08	—28°24	1.577
—0.530	1.207	—0.640	1.366	1.502	41°45	—27°56	1.646
—0.523	1.219	—0.634	1.374	1.515	41°23	—27°28	1.719
—0.517	1.231	—0.627	1.382	1.528	40°51	—27°01	1.796
—0.512	1.243	—0.621	1.390	1.541	40°24	—26°33	1.881
—0.509	1.255	—0.615	1.397	1.553	40°06	—26°06	1.969
—0.506	1.267	—0.609	1.406	1.567	39°38	—25°40	2.066
—0.504	1.279	—0.603	1.414	1.581	39°12	—25°14	2.168
—0.502	1.296	—0.594	1.425	1.599	38°45	—24°38	2.324

TABLE 5.

Flow *III*, $\frac{u_0}{a_*} = 1.496$, $\vartheta_0 = 40°$

x/r	u/a_*	v/a_*	λ	M	α	ϑ	$\bar{r}_2$
−2.304	1.718	0.510	1.791	2.399	24°39	16°33	1.0
−2.372	1.714	0.501	1.786	2.379	24°51	16°18	1.015
−2.438	1.710	0.492	1.779	2.362	25°06	16°02	1.031
−2.503	1.706	0.482	1.773	2.346	25°13	15°46	1.049
−2.565	1.702	0.472	1.766	2.326	25°23	15°29	1.068
−2.625	1.698	0.461	1.759	2.307	25°36	15°12	1.090
−2.682	1.694	0.451	1.753	2.291	25°51	14°54	1.113
−2.736	1.690	0.440	1.746	2.273	26°07	14°35	1.138
−2.787	1.686	0.429	1.740	2.257	26°19	14°16	1.166
−2.834	1.682	0.418	1.733	2.239	26°30	13°57	1.197
−2.878	1.678	0.406	1.726	2.221	26°49	13°37	1.230
−2.918	1.674	0.395	1.720	2.205	26°53	13°17	1.266
−2.953	1.670	0.383	1.713	2.188	27°12	12°55	1.308
−2.984	1.666	0.371	1.707	2.173	27°28	12°34	1.351
−3.011	1.662	0.359	1.700	2.156	27°35	12°12	1.400
−3.032	1.658	0.347	1.694	2.141	27°50	11°50	1.453
−3.049	1.654	0.335	1.687	2.124	28°10	11°27	1.512
−3.060	1.650	0.323	1.681	2.109	28°18	11°04	1.578
−3.066	1.646	0.311	1.675	2.098	28°28	10°41	1.650
−3.067	1.644	0.305	1.672	2.090	28°34	10°32	1.683

all the magnitudes are referred to the critical speed of sound a_* (local $M = 1$), for instance $U_1 = u_1/a_*$. In Tables 1-5 λ represents the dimensionless velocity, M the Mach number, α the Mach angle, θ the angular direction of the flow, r_2 dimensionless radius of family *II*. An example of combining conical flows and isentropic flows is given in Fig. 4.

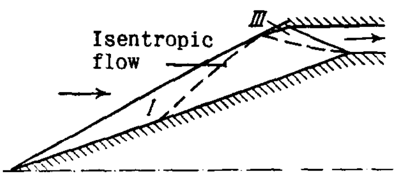

Fig. 4.

Analogous self-similar solutions of uniform unsteady flows have been examined by the author and co-workers [5].

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